JACOB LEVITZKI 1904–1956

Prof. Jacob Levitzki was born in Harson (Ukraine, USSR) in 1904 and died in Jerusalem in 1956. As a child he immigrated to Israel and later studied at Göttingen where he received his Ph.D. (1929). For short periods of time he was an assistant at the University of Kiel (Germany) and a Research Associate at Yale University (USA). He then returned to Israel and joined the Faculty of Sciences of the Hebrew University of Jerusalem. In 1954 he was awarded the Israel Prize for Exact Sciences.

Levitzki actually went to Gottingen with the intention of studying chemistry, but was persuaded by a friend, the late Prof. Benjamin Amirà, to listen to one of Emmy Noether's lectures and from that moment never left mathematics. (His son returned to chemistry.) He joined the large circle of the founders and creators of modern Ring Theory of the twenties and thirties. His work inspires young algebraists even today.

The beginning of his research was in the period when finiteness in algebra was being replaced by chain conditions and the first steps in extending Wedderburn Structure theorems of associative algebras met with the wall of nilpotent elements. It is not surprising that Levitzki's first interest was nilpotent elements and nil subrings. One of his well known result in this area is the nilpotency of the nil subrings of matrix rings [2] and of nil ideals in rings with ascending chain conditions [37], a result which since has been a source for many generalizations. His work on nilpotent elements [6] played a role in developing the now trivial notion of a rank of a matrix over non-commutative fields [5] and also led to a different notion of rank of elements in I-rings [35], which was not completely exploited because of his early death.

Artin's extension (1927) of Wedderburn Structure theorems to rings with ascending and descending chain conditions on one-sided ideals left open the problem of necessity of both conditions. Only in 1938–39 did both Hopkins (using left ideals and Levitzki (with right ideals) simultaneously and independently, in two different parts of the world, prove the now classic structure theorems of rings with the descending chain condition. It was unfortunate for Levitzki that his paper was sent to the editor of a European journal and the situation in Germany and the Second World War prevented his work from being presented to the world until much later. He subsequently gave other modifications of the chain conditions [15].

The next period of ring theory was faced with the challenge of avoiding chain conditions in the structure theorems, and here the first obstacle was the radical.

That was the period when the famous Koethe's problem on the nillity of sum of onesided nil ideals was formulated (and which is still unsolved). The Levitzki's Radical [10], originally known as the semi-nilpotent radical, proved to be the first useful radical, and contributed also to the penetration of the use of local properties in ring theory. Another of Levitzki's results worth mentioning in this area is the equality of the Prime and the Lower Radicals (proved independently by Nagata in Japan). Some of his ideas like T-nilpotency [31] were rediscovered years later in different contexts.

Then the period of bridging the gap between chain and no chain conditions came to ring theory. Various types of rings have been introduced in algebra and his paper on algebraic algebras [29, 33] and P-soluble rings [32] contain many interesting results and ideas that remain to be exploited further. Another direction was the problem of polynomial identities in rings (which first appeared in a paper by Dehn 1921 in connection with geometry). A theorem of Levitzki on local nilpotency of nil rings of bounded index [16] together with other results of M. Hall and N. Jacobson led to the reintroducing of the subject by Kaplansky in 1948. This was followed and pushed further by Levitzki [20]. Then came the famous result on the standard identity of matrix rings over commutative rings [21, 23] to which he was the major contributor.

Finally, we mention the special case of the Kurosch problem which was first formulated by Levitzki and known as such: "Are there finitely generated nil rings which are not nilpotent?" A question which seems to be the major obstacle in developing a Ring Theory.

Levitzki was known as an excellent teacher, and he had a feeling for advances of modern mathematics. This led him to introduce vector spaces, modules and linear transformation in the first year courses (undergraduate) in algebra at the Hebrew University as early as 1946. His students heard him saying that these notions and notations will prevail in modern mathematics and the young generation of mathematicians must encounter them as early as possible—and this he turned into practice.

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